

**RELATIVE PHASES IN  $D^0 \rightarrow K^0 K^- \pi^+$   
AND  $D^0 \rightarrow \bar{K}^0 K^+ \pi^-$  DALITZ PLOTS**Bhubanjyoti Bhattacharya and Jonathan L. Rosner  
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The processes  $D^0 \rightarrow K^0 K^- \pi^+$  and  $D^0 \rightarrow \bar{K}^0 K^+ \pi^-$  involve intermediate vector resonances whose amplitudes and phases are related to each other via flavor SU(3) symmetry. A closer look at Dalitz plots for these two processes is expected to shed light on our understanding of the usefulness of flavor SU(3) symmetry in studying charm decays. In the present work we use data from the BaBar Collaboration's publication in 2002. The goal is to reproduce Dalitz plot amplitudes and phases assuming flavor SU(3) symmetry and compare them with experiment.

While an SU(3) fit is able to account for the relative magnitudes of the amplitudes for the decays  $D^0 \rightarrow K^{*-} K^+$  and  $D^0 \rightarrow K^{*+} K^-$ , the current BaBar sample (based on an integrated luminosity of only  $22 \text{ fb}^{-1}$ ) provides only  $1-2\sigma$  evidence for the decays  $D^0 \rightarrow K^{*0} \bar{K}^0$  and  $D^0 \rightarrow \bar{K}^{*0} K^0$ , with magnitudes and phases not in accord with predictions. The CLEO collaboration could potentially produce an analysis using data with better statistics, and an analysis based on the full BaBar sample (more than 20 times the 2002 value) should definitively settle the question.

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## I Introduction

An important contribution to the decay processes  $D^0 \rightarrow 3P$ , where  $P$  represents a pseudoscalar meson, involves the intermediate step in which the  $D$  meson first decays into a  $P$  and a vector meson ( $V$ ). The vector meson then decays into two pseudoscalars. In general, in a decay with three final  $P$  states the combination of any pair of final pseudoscalars may result from the decay of a  $V$  as long as charge, isospin, strangeness, etc. are conserved. Evidence of formation of such resonances is seen in Dalitz plots as bands of events corresponding to the invariant mass-squared of the pair of final state  $P$  mesons. As such, they provide information about the amplitude and phase for the process  $D \rightarrow PV$ . Overlapping vector resonance bands on Dalitz plots interfere according to their relative phases.

Amplitudes and phases of  $D \rightarrow PV$  decays were studied in detail using an SU(3) flavor symmetry formalism in Ref. [1]. Relative phase relations based on SU(3) flavor symmetry were exploited in Refs. [2, 3, 4] to observe the successes of the flavor SU(3) symmetry formalism in predicting interferences on several  $D \rightarrow 3P$  Dalitz plots. In the present article we consider the Dalitz plots for  $D^0 \rightarrow K^0 K^- \pi^+$  and  $D^0 \rightarrow \bar{K}^0 K^+ \pi^-$ . We

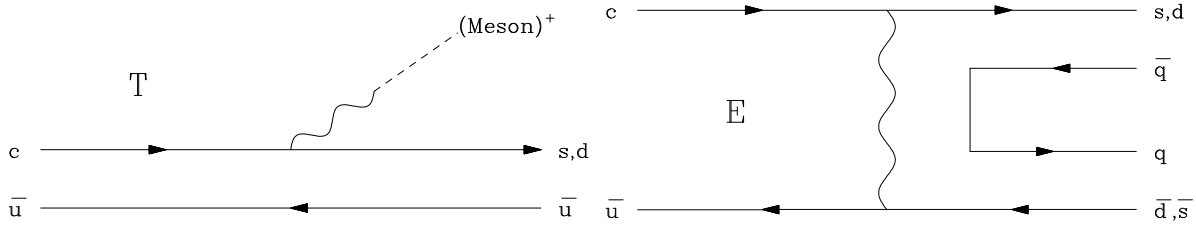


Figure 1: Graphs describing tree ( $T$ ) and exchange ( $E$ ) amplitudes

extract amplitudes and phases for the relevant  $D \rightarrow PV$  intermediate processes from data published by the BaBar collaboration [5], and compare them with theoretical predictions using flavor SU(3) symmetry.

We briefly review the flavor SU(3) symmetry technique in Sec. II. In Sec. III we quote the theoretical results for relevant  $D \rightarrow PV$  processes, and compare them with data in Sec. IV. We conclude in Sec. V.

## II Review of flavor SU(3) symmetry technique

The flavor symmetry approach to be used here was discussed in detail in [1]. Here we recall the basic points. We denote the relevant Cabibbo-favored (CF) amplitudes, proportional to the product  $V_{ud}V_{cs}^*$  of Cabibbo-Kobayashi-Maskawa (CKM) factors, by amplitudes labeled as  $T$  (“tree”) and  $E$  (“exchange”), illustrated in Fig. 1. The singly-Cabibbo-suppressed (SCS) amplitudes, proportional to the product  $V_{us}V_{cs}^*$  or  $V_{ud}V_{cd}^*$ , are then obtained by using the ratio  $\text{SCS}/\text{CF} = \tan \theta_C \equiv \lambda = 0.2305$  [6], with  $\theta_C$  the Cabibbo angle and signs governed by the relevant CKM factors. The subscript  $P$  or  $V$  on an amplitude denotes the meson ( $P$  or  $V$ ) containing the spectator quark in the  $PV$  final state. The partial width  $\Gamma(H \rightarrow PV)$  for the decay of a heavy meson  $H$  is given in terms of an invariant amplitude  $\mathcal{A}$  as:

$$\Gamma(H \rightarrow PV) = \frac{p^{*3}}{8\pi M_H^2} |\mathcal{A}|^2 \quad (1)$$

where  $p^*$  is the center-of-mass (c.m.) 3-momentum of each final particle, and  $M_H$  is the mass of the decaying heavy meson. With this definition the amplitudes  $\mathcal{A}$  are dimensionless.

The amplitudes  $T_V$  and  $E_P$  were obtained from fits to rates of CF  $D \rightarrow PV$  decays not involving  $\eta$  or  $\eta'$  [1]. To specify the amplitudes  $T_P$  and  $E_V$ , however, one needs information on the  $\eta - \eta'$  mixing angle ( $\theta_\eta$ ). Table I summarizes these results for two values  $\theta_\eta = 19.5^\circ$  and  $11.7^\circ$ .

## III Relevant $D^0 \rightarrow PV$ processes

In Tables II and III we list the  $D^0 \rightarrow PV$  amplitudes relevant in Dalitz plots of interest for  $\theta_\eta = 19.5^\circ$  and  $\theta_\eta = 11.7^\circ$ , respectively. Also included are their representations and values in terms of flavor-SU(3) amplitudes.

Flavor SU(3) symmetry requires the amplitudes  $\mathcal{A}(D^0 \rightarrow K^{*0} \bar{K}^0)$  and  $\mathcal{A}(D^0 \rightarrow \bar{K}^{*0} K^0)$  to be equal in magnitude but  $180^\circ$  apart in phase. Since these two processes show up in two different Dalitz plots, this offers us a way to check the relative amplitudes and phases within individual Dalitz plots obtained from Dalitz plot fits. In the following section we obtain the

Table I: Solutions for  $T_V$ ,  $E_P$ ,  $T_P$  and  $E_V$  amplitudes in Cabibbo-favored charmed meson decays to  $PV$  final states, for  $\eta$ - $\eta'$  mixing angles of  $\theta_\eta = 19.5^\circ$  and  $11.7^\circ$ .

$PV$ ampl.	$\theta_\eta = 19.5^\circ$		$\theta_\eta = 11.7^\circ$	
	Magnitude ( $10^{-6}$ )	Relative strong phase	Magnitude ( $10^{-6}$ )	Relative strong phase
$T_V$	$3.95 \pm 0.07$	Assumed 0	These results are independent of $\theta_\eta$	
$E_P$	$2.94 \pm 0.09$	$\delta_{E_P T_V} = (-93 \pm 3)^\circ$		
$T_P$	$7.46 \pm 0.21$	Assumed 0	$7.69 \pm 0.21$	Assumed 0
$E_V$	$2.37 \pm 0.19$	$\delta_{E_V T_V} = (-110 \pm 4)^\circ$	$1.11 \pm 0.22$	$\delta_{E_V T_V} = (-130 \pm 10)^\circ$

Table II: Amplitudes for  $D^0 \rightarrow PV$  decays of interest for the present discussion (in units of  $10^{-6}$ ). Here we have taken  $\theta_\eta = 19.5^\circ$ .

Dalitz plot	$D^0$ final state	Amplitude representation	Amplitude $A$			
			Re	Im	$ A $	Phase ( $^\circ$ )
$D^0 \rightarrow K^0 K^- \pi^+$	$K^{*+} K^-$	$\lambda(T_P + E_V)$	1.533	-0.513	1.616	-18.5
	$\bar{K}^{*0} K^0$	$\lambda(E_V - E_P)$	-0.151	0.163	0.223	132.8
$D^0 \rightarrow \bar{K}^0 K^+ \pi^-$	$K^{*-} K^+$	$\lambda(T_V + E_P)$	0.875	-0.677	1.106	-37.7
	$K^{*0} \bar{K}^0$	$\lambda(E_P - E_V)$	0.151	-0.163	0.223	-47.2

amplitudes and phases for the above amplitudes using Dalitz plot fit fractions and relative phases and compare these results with theoretical predictions using flavor SU(3) symmetry.

## IV Comparison of data with theoretical predictions

In order to obtain amplitudes and phases for the amplitudes mentioned in the previous section from Dalitz plot fit fractions, one needs to keep in mind that the  $D \rightarrow PV$  process is an intermediate to the complete 3 body decay  $D \rightarrow 3P$ . The Dalitz plot fit fractions also contain some information about the vector meson decay and this needs to be factored

Table III: Amplitudes for  $D^0 \rightarrow PV$  decays of interest for the present discussion (in units of  $10^{-6}$ ). Here we have taken  $\theta_\eta = 11.7^\circ$ .

Dalitz plot	$D^0$ final state	Amplitude representation	Amplitude $A$			
			Re	Im	$ A $	Phase ( $^\circ$ )
$D^0 \rightarrow K^0 K^- \pi^+$	$K^{*+} K^-$	$\lambda(T_P + E_V)$	1.608	-0.196	1.620	-6.9
	$\bar{K}^{*0} K^0$	$\lambda(E_V - E_P)$	-0.129	0.481	0.498	105.0
$D^0 \rightarrow \bar{K}^0 K^+ \pi^-$	$K^{*-} K^+$	$\lambda(T_V + E_P)$	0.875	-0.677	1.106	-37.7
	$K^{*0} \bar{K}^0$	$\lambda(E_P - E_V)$	0.129	-0.481	0.498	-75.0

Table IV: Conventions for the order of two pseudoscalar mesons in vector meson decay and associated Clebsch-Gordan factors assuming the cyclic convention of Ref. [7].

Dalitz Plot	Bachelor Particle		Vector Meson Decay			$p^*$ (in MeV)
	Meson	Index	Process	Indices	Clebsch Factor	
$D^0 \rightarrow K^0 K^- \pi^+$	$K^0$	1	$\bar{K}^{*0} \rightarrow K^- \pi^+$	23	$-\sqrt{2/3}$	605
	$K^-$	2	$K^{*+} \rightarrow \pi^+ K^0$	31	$\sqrt{2/3}$	610
	$\pi^+$	3	—	—	—	—
$D^0 \rightarrow \bar{K}^0 K^+ \pi^-$	$\bar{K}^0$	1	$K^{*0} \rightarrow K^+ \pi^-$	23	$\sqrt{2/3}$	605
	$K^+$	2	$K^{*-} \rightarrow \pi^- \bar{K}^0$	31	$-\sqrt{2/3}$	610
	$\pi^-$	3	—	—	—	—

out in order for any comparison with theoretical predictions from the flavor-SU(3) method. This, however, is fairly simple since the fraction of a vector meson's decay amplitude to a pair of  $P$  mesons can be given by the relevant isospin Clebsch-Gordan factor.

To obtain the correct Clebsch-Gordan factor, one notes that the spin part of the amplitude for the process  $D \rightarrow RC \rightarrow ABC$  ( $R$  represents the intermediate resonance while  $A$ ,  $B$  and  $C$  are the final state pseudoscalar mesons) is proportional to the product  $\vec{p}_A \cdot \vec{p}_C$  ( $\vec{p}_i$  is the 3-momentum of the final state particle  $i$  in the rest frame of  $R$ .) Since the particles  $A$  and  $B$  have equal and opposite 3-momenta in the resonance rest frame, this implies then that swapping  $A$  and  $B$  while calculating the amplitude would result in an additional phase difference of  $\pi$ . It is thus important to know the phase convention used to obtain the amplitudes. In the present case, due to the absence of a stated phase convention in Ref. [5], we assume a cyclic permutation convention often employed by the BaBar Collaboration elsewhere [7]. This convention is presented in Table IV. Using this convention one may then calculate the appropriate isospin Clebsch-Gordan coefficients, also noted in Table IV.

The phase space factors for the two  $D \rightarrow PV$  processes from each Dalitz plot are not the same since the vector mesons involved have slightly different masses. This very small difference, noted in Table IV, has been neglected.

Using the appropriate Clebsch-Gordan coefficients we now translate the fit parameters into amplitudes and phases that can be compared with theoretical predictions. In Table V we quote the fit fractions and phases from a fit to BaBar data [5] for relevant intermediate  $D^0 \rightarrow PV$  decays corresponding to each Dalitz plot. Fit fractions quoted in Table V are normalized so as to represent percentage of each decay mode in the specific Dalitz plots. This normalization is different for the two different Dalitz plots. In order to compare amplitudes for  $D \rightarrow PV$  processes from two different Dalitz plots it is useful to choose a universal normalization. To achieve this we make use of the total branching fraction for the  $D \rightarrow 3P$  process for each Dalitz plot, so as to calculate the fraction of each  $D \rightarrow PV$  process relative to a common rate or amplitude. In Table V, in addition to the above data, we also quote the total branching fractions for the overall  $D^0 \rightarrow 3P$  process in each Dalitz plot, relative to the process  $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$ .

We make use of the BaBar data [5] quoted in Table V to calculate the relative amplitudes and phases of the relevant  $D \rightarrow PV$  decays. The magnitudes and phases of the amplitudes

Table V: Dalitz plot fit to data from the BaBar collaboration [5].

Dalitz Plot	Branching Fraction (%) (rel to $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$ )	$D^0$ final state	Experiment	
			Fit Fraction (%)	Phase ( $^\circ$ )
$D^0 \rightarrow K^0 K^- \pi^+$	$8.32 \pm 0.29 \pm 0.56$	$K^{*+} K^-$	$63.6 \pm 5.1 \pm 2.6$	0 (fixed)
		$\bar{K}^{*0} K^0$	$0.8 \pm 0.5 \pm 0.1$	$175 \pm 22$
$D^0 \rightarrow \bar{K}^0 K^+ \pi^-$	$5.68 \pm 0.25 \pm 0.41$	$K^{*-} K^+$	$35.6 \pm 7.7 \pm 2.3$	0 (fixed)
		$K^{*0} \bar{K}^0$	$2.8 \pm 1.4 \pm 0.5$	$-126 \pm 19$

Table VI: Amplitudes for  $D^0 \rightarrow PV$  decays from Dalitz plots of interest for the present discussion (in units of  $10^{-6}$ ). Here we have taken  $\theta_\eta = 19.5^\circ$ , and  $\lambda = 0.2305$  [6]. The experimental amplitudes have been taken from BaBar data [5] and have a normalization such that the largest amplitude is fixed to 1. The phases in lines 2 and 3 of the last column have been flipped by  $180^\circ$  in comparison with those listed in lines 2 and 3 of Table V to correspond to the negative signs of the Clebsch-Gordan coefficients in Table IV.

$D^0$ final state	Amplitude Representation	Theory		Experiment	
		Amplitude	Phase ( $^\circ$ )	Amplitude	Phase ( $^\circ$ )
$K^{*+} K^-$	$\lambda(T_P + E_V)$	$1.616 \pm 0.060$	$-18.5 \pm 1.6$	1 (fixed)	0 (fixed)
$\bar{K}^{*0} K^0$	$\lambda(E_V - E_P)$	$0.223 \pm 0.050$	$132.8 \pm 13.0$	$0.112^{+0.032}_{-0.045}$	$-5 \pm 22$
$K^{*-} K^+$	$\lambda(T_V + E_P)$	$1.106 \pm 0.033$	$-37.7 \pm 1.5$	$0.618^{+0.078}_{-0.089}$	180 (fixed)
$K^{*0} \bar{K}^0$	$\lambda(E_P - E_V)$	$0.223 \pm 0.050$	$-47.2 \pm 13.0$	$0.173^{+0.042}_{-0.057}$	$-126 \pm 19$

are obtained relative to that of the process  $D^0 \rightarrow K^{*+} K^-$  with maximum amplitude. These results are listed under the last two columns in Table VI. In Table VI we also list the predictions of amplitudes and phases for the same processes obtained using the flavor-SU(3)-symmetry technique.

The ratio of the amplitude  $|\mathcal{A}(D^0 \rightarrow K^{*-} K^+)|$  relative to  $|\mathcal{A}(D^0 \rightarrow K^{*+} K^-)|$  is predicted to be equal to a corresponding ratio of Cabibbo-favored amplitudes:

$$\frac{|\mathcal{A}(D^0 \rightarrow K^{*-} K^+)|}{|\mathcal{A}(D^0 \rightarrow K^{*+} K^-)|} = \frac{|\mathcal{A}(D^0 \rightarrow K^{*-} \pi^+)|}{|\mathcal{A}(D^0 \rightarrow \rho^{*+} K^-)|}. \quad (2)$$

The left-hand side is  $0.618^{+0.078}_{-0.089}$  as computed using the experimental numbers from Table VI. On the other hand, the right-hand side is  $0.685 \pm 0.032$ , when computed from the respective Cabibbo-favored amplitudes [1]. Thus flavor SU(3) symmetry seems to be obeyed for the dominant  $VP$  sub-amplitudes in  $D^0 \rightarrow K_S K^\pm \pi^\mp$ .

Flavor SU(3) predicts equal magnitudes for the much smaller amplitudes  $\mathcal{A}(D^0 \rightarrow \bar{K}^{*0} K^0)$  and  $\mathcal{A}(D^0 \rightarrow K^{*0} \bar{K}^0)$ . The central values of the magnitudes obtained from the experimental fit are respectively smaller and larger than the theoretical prediction:

$$\frac{|\mathcal{A}(D^0 \rightarrow \bar{K}^{*0} K^0)|}{|\mathcal{A}(D^0 \rightarrow K^{*+} K^-)|} = 0.112^{+0.032}_{-0.045} \text{ (expt.) } vs. 0.138 \pm 0.033 \text{ (theory)}; \quad (3)$$

$$\frac{|\mathcal{A}(D^0 \rightarrow \bar{K}^0 K^{*0})|}{|\mathcal{A}(D^0 \rightarrow K^{*+} K^-)|} = 0.173_{-0.057}^{+0.042} \text{ (expt.) } \textit{vs.} \ 0.138 \pm 0.033 \text{ (theory)} . \quad (4)$$

In the above equations we only quote the  $1\sigma$  error bars for the ratios. Although the probability distribution functions for the input branching fractions may be taken to be Gaussian, the probability distribution for the amplitude ratios (expt.) are quite different. In fact one may check that the ratios (expt.) in Eqns. (3) and (4) are consistent with zero at the  $1.55\sigma$  and  $1.81\sigma$  levels, respectively.

Given the large experimental errors, the discrepancies in magnitudes from the flavor-SU(3) predictions are not yet evidence for violation of this symmetry. However, relative phases of the relevant amplitudes obtained from theory and experiment are not in agreement, the discrepancies being  $(156 \pm 22)^\circ$  for  $D^0 \rightarrow K^0 K^- \pi^+$  and  $(64 \pm 19)^\circ$  for  $D^0 \rightarrow \bar{K}^0 K^+ \pi^-$ , quoting only the experimental error. Similar conclusions follow from the predictions for  $\theta_\eta = 11.7^\circ$ . This could arise from a misunderstood convention for the vector meson decay, as explained in the previous section, or could signal a breakdown of the flavor-SU(3) approach. If the latter, it would be the first such instance, as earlier analyses [2, 3, 4] reproduced such relative phases successfully. One may also argue that the experimental relative phases and the error bars on them are meaningless, since the corresponding amplitude ratios are consistent with zero. In that case one needs a larger data sample to identify the correct relative phases and corresponding error bars.

We conclude for a number of reasons that it is premature to declare flavor SU(3) invalid for the decays in question: (1) The BaBar  $K^{*0} \bar{K}^0$  and  $\bar{K}^{*0} K^0$  amplitudes are marginal and have not yet been confirmed by any other experiment; (2) BaBar's phase convention has not been explicitly stated and, though requested, has not been made available; (3) results from CLEO with a larger data sample will soon be available; and (4) BaBar's total sample is more than 20 times as large and an updated analysis would provide much more convincing statistics.

## V Conclusions

Flavor SU(3) has had notable success in predicting relative phases of Dalitz plot amplitudes in charm [2, 3, 4] and beauty [8] decays. Nevertheless, the pattern of interference between vector meson resonance bands in  $D^0 \rightarrow K^0 K^- \pi^+$  and  $D^0 \rightarrow \bar{K}^0 K^+ \pi^-$  Dalitz plots, based on analysis of a small fraction of currently available data [5], is at odds with SU(3) predictions. Notably, (1) the relative phase between  $K^{*+} K^-$  and  $\bar{K}^{*0} K^0$  bands in  $D^0 \rightarrow K^0 K^- \pi^+$  differs from the SU(3) prediction by  $(156 \pm 22)^\circ$ , where we quote only the experimental error; (2) the relative phase between vector meson resonance bands in  $D^0 \rightarrow \bar{K}^0 K^+ \pi^-$  differs from the SU(3) prediction by  $(64 \pm 19)^\circ$ ; (3) the ratio  $|\mathcal{A}(D^0 \rightarrow \bar{K}^{*0} K^0)/\mathcal{A}(D^0 \rightarrow K^{*0} \bar{K}^0)|$ , predicted to be 1, is  $0.64 \pm 0.27$ .

In view of the facts that the BaBar analysis [5] employed an integrated luminosity of only  $22 \text{ fb}^{-1}$ , whereas a data sample of more than twenty times that is now available, and that the CLEO Collaboration also has a large sample of such events, this Dalitz plot would appear to be a prime target for re-analysis.

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